AP Calculus AB
Test: The Integral and the Fundamental Theorem of Calculus
Instructor's Version

Free-Response Question 1 — no calculator (15 points)

1. The velocity of a car is recorded every 10 seconds, as shown in the table below.

t (sec)	0	10	20	30	40	50	60	70
$v(t)$ (ft/sec)	4	12	22	38	50	57	59	60

A. Explain the meaning of $\int_{10}^{50} v(t)dt$ in terms of the car's travel. Use correct units. (3 points)

Answer:
The equation $v(t) > 0$ on the interval $10 \le t \le 50$, $\int_{10}^{50} v(t)dt$ represents the distance, in feet, that the car traveled from 10 seconds to 50 seconds.

1 point: correct units
1 point: distance
1 point: time interval

B. Evaluate $\int_{20}^{60} v(t)dt$ using a left-hand approximation with n = 4. (3 points)

Answer:
$\int_{20}^{60} v(t)dt = 10(22 + 38 + 50 + 57) = 1670$ feet

1 point: 10(22 + 38 + 50 + 57)
1 point: answer
1 point: correct units

C. Evaluate $\int_{0}^{60} v(t)dt$ using a midpoint approximation with n = 3. (3 points)

Answer:
$\int_{0}^{60} v(t)dt = 20(12 + 38 + 57) = 2140$ feet

1 point: 20(12 + 38 + 57)
1 point: answer
1 point: correct units

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D. Evaluate $\int_0^{70} v(t)dt$ using a trapezoidal approximation with n = 7. (3 points)

Answer:

$$\int_0^{70} v(t)dt = \frac{1}{2}(10)\left[4 + 2(12 + 22 + 38 + 50 + 57 + 59) + 60\right] = 2700 \text{ feet}$$

1 point: correct formula/setup of problem
1 point: answer
1 point: correct units

E. Determine the average acceleration of the car over the time interval $0 \le t \le 70$. (3 points)

Answer:

$$\frac{v(70) - v(0)}{70 - 0} = \frac{60 - 4}{70} = \frac{4}{5} \text{ or } 0.8 \text{ ft/sec}^2$$

1 point: $\frac{v(70) - v(0)}{70 - 0}$
1 point: answer
1 point: correct units

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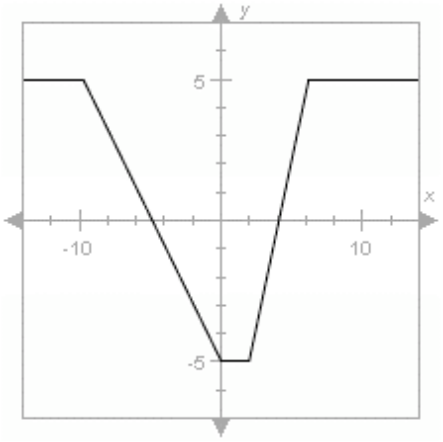
Free-Response Question 2 — with calculator (15 points)

2. The graph of the function *f* is shown below with domain [-12, 10]. Let $g(x) = \int_{-12}^{x} f(t)dt$.



A. Evaluate $g(0)$, $g'(0)$, and $g''(0)$. (6 points)

Answer:

$$g(0) = \int_{-12}^{0} f(t)dt = 10 + \frac{25}{2} - \frac{25}{2} = 10$$

1 point: $\int_{-12}^{0} f(t)dt$
1 point: answer

$g'(x) = f(x)$, therefore $g'(0) = f(0) = -5$

1 point: $g'(x) = f(x)$
1 point: answer

$g''(x) = f'(x)$, therefore $g''(0) = f'(0)$, which does not exist because *f* is not differentiable at 0.

1 point: $g''(x) = f'(x)$
1 point: $f'(0)$ d.n.e with reasoning — student may say that *f* is not differentiable at 0 because the graph of *f* has a cusp when *x* = 0

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B. Find all possible values of *x* such that $g(x) = 0$. Justify your answer. (5 points)

Answer: $g(x) = 0$ implies $\int_{-12}^{x} f(t)dt = 0$ or that the signed area from -12 to *x* is zero. Signed area from -12 to -5 is 22.5 square units. Signed area from -5 to 2 is -22.5 square units. Signed area from -12 to 4 is -10 square units, and signed area from 4 to 6 is +10 square units.

Thus, $g(x) = 0$ when $x = 2$ or $x = 6$.

2 points: answers
3 points: reasoning

C. Determine where the graph of $g(x)$ has horizontal tangent lines, if anywhere. Justify your answer. (4 points)

Answer: $g'(x) = f(x) = 0$

2 points: sets derivative equal to 0

$f(x) = 0$ when $x = 5$ or $x = -5$
g has horizontal tangents at $x = 5$ and at $x = -5$

2 points: 1 for each answer

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